

Investigation: Congruent and Similar Triangles

Name: _____

Key terms

Two shapes are **congruent** (\cong) if they are both the same _____ and _____.

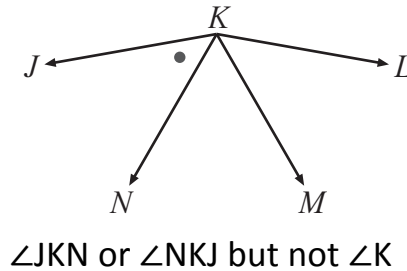
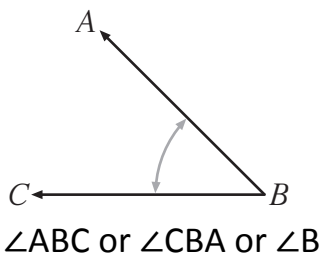
Two shapes are **similar** (\sim) if they are the same _____ but not the same _____.

Naming lines, angles and triangles

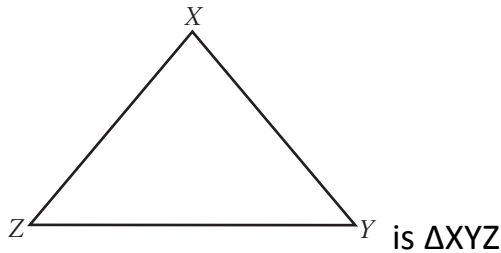
- Name **lines** using the letters of the two points (or vertices) the line is between:



- Name **angles** using the letters involved, with the letter at the vertex in the middle.

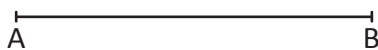


- Name **triangles** using the letters of the three points involved:

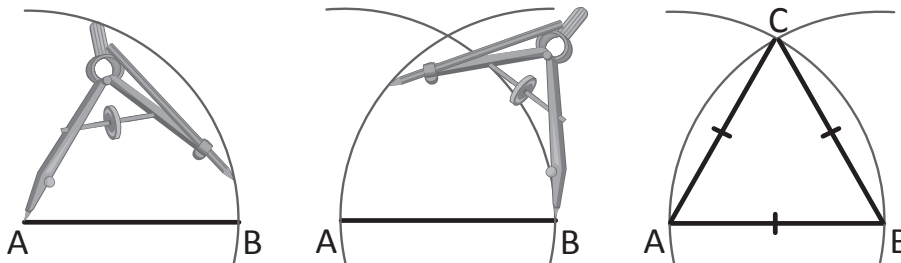


Drawing triangles

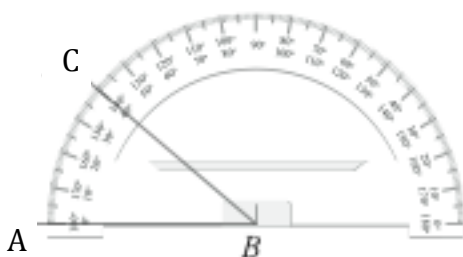
- Most triangles will start with drawing a line of a known length. Use a ruler:



- To draw lines of a known length but at an unknown angle, use a compass:



- To draw lines at a known angle, use a protractor to mark the angle before ruling:



If **one side** of a triangle is known, how many different triangles can be drawn?

1. Draw $\triangle ABC$ using the following measurements:

$AB = 8\text{cm}$

2. Now try to draw a different triangle $\triangle DEF$ using the following measurements:

$DE = 8\text{cm}$

3. Was it possible to draw a different shape or size of triangle while keeping one side the same? Why or why not?

If **two sides** of a triangle are known, how many different triangles can be drawn?

1. Draw $\triangle GHI$ using the following measurements:

$GH = 9\text{cm}$

$HI = 5\text{cm}$

2. Now try to draw a different triangle $\triangle JKL$ using the following measurements:

$JK = 9\text{cm}$

$KL = 5\text{cm}$

3. Was it possible to draw a different shape or size of triangle while keeping two sides the same? Why or why not?

If **three sides** of a triangle are known, how many different triangles can be drawn?

1. Draw $\triangle MNO$ using the following measurements:

$$MN = 10\text{cm}$$

$$NO = 8\text{cm}$$

$$MO = 6\text{cm}$$

2. Is it possible to draw this triangle any other way? Why or why not?

3. Now try to draw a different triangle $\triangle PQR$ using the following measurements:

$$PQ = 5\text{cm}$$

$$QR = 4\text{cm}$$

$$PR = 3\text{cm}$$

4. What do you notice about $\triangle MNO$ and $\triangle PQR$?

If **one angle** of a triangle is known, how many different triangles can be drawn?

1. Draw $\triangle STU$ using the following measurements:

$$ST = 5\text{cm}$$

$$\angle STU = 60^\circ$$

$$TU = 4\text{cm}$$

2. Is it possible to draw this triangle any other way? Why or why not?

3. Would the triangle be different if the two side lengths were not given?

4. Now try to draw a different triangle $\triangle VWX$ using the following measurements:

$$VW = 10\text{cm}$$

$$\angle VWX = 60^\circ$$

$$WX = 8\text{cm}$$

5. What do you notice about $\triangle STU$ and $\triangle VWX$? Why does the angle need to be the same?

If **two sides** of a **right-angled triangle** are known, how many different triangles can be drawn?

The longest side of a right-angled triangle is called the **hypotenuse** and is always opposite the right angle. This situation is different to the last where the known angle was between the two known sides.

1. Draw the right-angled triangle, ΔYZA , using the following measurements:

YZ = 10cm (hypotenuse)

ZA = 6cm

2. Is it possible to draw this triangle any other way? Why or why not?

If **two angles** of a triangle are known, how many different triangles can be drawn?

1. Draw $\triangle BCD$ using the following measurements:

$$BC = 8\text{cm}$$

$$\angle CBD = 60^\circ$$

$$\angle BCD = 30^\circ$$

2. Is it possible to draw this triangle any other way? Why or why not?

3. Now try to draw a different triangle $\triangle EFG$ using the following measurements:

$$EF = 10\text{cm}$$

$$\angle FEG = 60^\circ$$

$$\angle EFG = 30^\circ$$

4. What do you notice about $\triangle BCD$ and $\triangle EFG$?

5. Why is knowing two angles in a triangle the same as knowing three angles?

Summary

Two triangles are **congruent** if all matching (or corresponding) sides and angles are _____.

If triangles can only be drawn one way based on the measurements given, then all those triangles will be _____.

Two triangles are **similar** if all matching angles are _____ and all matching sides are the same _____.

If all the triangles drawn, based on the measurements given, are the same shape but different sizes, then all those triangles are _____.

Once you have completed the investigation, complete the summary table below. In each case, decide whether the number of corresponding sides or angles given is enough to prove that the triangles are congruent or similar.

Corresponding sides or angles that are equal or same ratio	Congruent	Similar
One side (S)	<input type="checkbox"/>	<input type="checkbox"/>
Two sides (SS)	<input type="checkbox"/>	<input type="checkbox"/>
Three sides (SSS)	<input type="checkbox"/>	<input type="checkbox"/>
One angle (A)	<input type="checkbox"/>	<input type="checkbox"/>
One angle and one side (AS)	<input type="checkbox"/>	<input type="checkbox"/>
One angle between two sides (SAS)	<input type="checkbox"/>	<input type="checkbox"/>
One right angle, hypotenuse and one other side (RHS)	<input type="checkbox"/>	<input type="checkbox"/>
Two angles (AA)	<input type="checkbox"/>	<input type="checkbox"/>
Two angles and one side (ASA)	<input type="checkbox"/>	<input type="checkbox"/>