Probability

PROBABILITY

Series I

MATHLETICS
www.mathletics.com.au

Theoretical probability

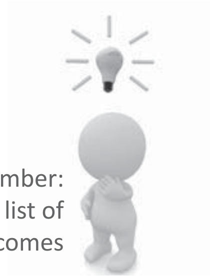
Theoretical probability is the expected chance of events written as fractions, decimals or percentages.

It compares how many times a particular event can happen with all the possible outcomes.

Theoretical probability of an event $P(E) = \dfrac{\text{Total number of favourable outcomes } n(E)}{\text{The total number of possible outcomes } n(S)}$

- $n(E) =$ the number of outcomes matching the result we are looking for.
- $n(S) =$ the total number of outcomes in the sample space.

Remember:
Sample space is a list of
all the possible outcomes



The total number of favourable outcomes can never be more than the total number of possible outcomes.

$\therefore$ The probability of an event can **only** be any value from 0 to 1.

$\therefore$ Probabilities in percentage form can **only** be any value from 0% to 100%.

Here are some typical questions

(i) A pencil case has three blue, one green, two pink and two yellow highlighters.

α) List the sample space for all the possible outcomes if one highlighter was picked out at random.

The sample space $S = \{Blue, Blue, Blue, Green, Pink, Pink, Yellow, Yellow\}$

β) Calculate $P(Blue)$ after first calculating $n(Blue)$ and $n(S)$.

$$n(Blue) = 3 \qquad n(S) = 8$$

$$\therefore P(Blue) = \frac{n(Blue)}{n(S)} = \frac{3}{8}$$ Probability as a fraction

$$= 0.375$$ Probability as a decimal

$$= 37.5\%$$ Probability as a percentage

(ii) If $P(E) = \frac{4}{5}$ and $n(S) = 15$, what is the value of $n(E)$?

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{5}$$ Simplified fraction

$$= \frac{n(E)}{15}$$ Equivalent fraction with denominator of 15

$$n(E) = 12$$ Numerator of simplified fraction $\times 3$

Theoretical probability

1 Write down the sample space S and calculate $n(S)$ for each of these spinners:

a

$S =$ ______

$n(S) = n(Numbers) =$ ______

b

$S =$ ______

$n(S) = n(Colours) =$ ______

c

$S =$ ______

$n(S) = n(Options) =$ ______

2 Calculate $P(E)$ accurate to 2 decimal places for these favourable outcomes and sample set values.

a $n(E) = 2, \quad n(S) = 25$

$P(E) =$

b $n(White\ flowers) = 10, \quad n(Flowers) = 14$

$P(White\ flowers) =$

c $n(Brown) = 7, \quad n(Cards) = 16$

$P(Brown\ card) =$

d $n(Odd\ numbers) = 12, \quad n(Numbers) = 33$

$P(Odd\ numbers) =$

3 Calculate $P(E)$, as a percentage of these:

a $n(E) = 1, \quad n(S) = 2$

$P(E) =$

b $n(E) = 3, \quad n(S) = 25$

$P(E) =$

c $n(E) = 36, \quad n(S) = 48$

$P(E) =$

d $n(E) = 5, \quad n(S) = 8$

$P(E) =$

4 Calculate $n(E)$ or $n(S)$ for each of these:

a $P(E) = \frac{1}{4}, \quad n(S) = 12$

$\therefore n(E) =$

b $P(E) = 30\%$ (i.e. $\frac{3}{10}$), $\quad n(E) = 15$

$\therefore n(S) =$

c $P(Orange) = 0.6$ (i.e. $\frac{6}{10}$), $\quad n(Oranges) = 21$

$\therefore n(Fruit) =$

d $n(Animals) = 12, \quad P(Duck) = 75\%$

$\therefore n(Ducks) =$

Theoretical probability

5 A bag contains the following wooden tiles with numbers or letters in the quantities given.

5 $\times 2$ **9** $\times 3$ **2** $\times 2$ **6** $\times 4$ **K** $\times 1$ **H** $\times 3$ **B** $\times 2$ **T** $\times 3$

A wooden tile is drawn at random from the bag.

a Write down the total values for each of these favourable outcomes:

(i) $n(9) =$ ☐ (ii) $n(B) =$ ☐ (iii) $n(Letter) =$ ☐ (iv) $n(Number) =$ ☐

(v) $n(H$ or $6) =$ ☐ (vi) $n(Number < 9) =$ ☐ (vii) $n(7) =$ ☐ (viii) $n(Even) =$ ☐

b Write down the sample space for all the possible outcomes and the value of (S).

(i) $S =$

(ii) $n(S) =$ ☐

c Write these theoretical probabilities as simplified fractions if a tile is drawn at random from the bag.

(i) $P(9) = \frac{n(9)}{n(S)} = \frac{☐}{☐}$ (ii) $P(Even) = \frac{n(Even)}{n(S)} = \frac{☐}{☐}$ (iii) $P(B) = \frac{☐}{☐}$

(iv) $P(Number < 6) = \frac{☐}{☐}$ (v) $P(H$ or $B) = \frac{☐}{☐}$ (vi) $P(6$ or 9 or $T) = \frac{☐}{☐}$

6 A cup and ball guessing game is played with three cups and one ball. It is up to players to guess which cup has the ball underneath it after they have been shuffled around.

a Write down the sample space for all the possible outcomes when a cup is lifted up and the value of $n(S)$.

$S =$ $n(S) =$

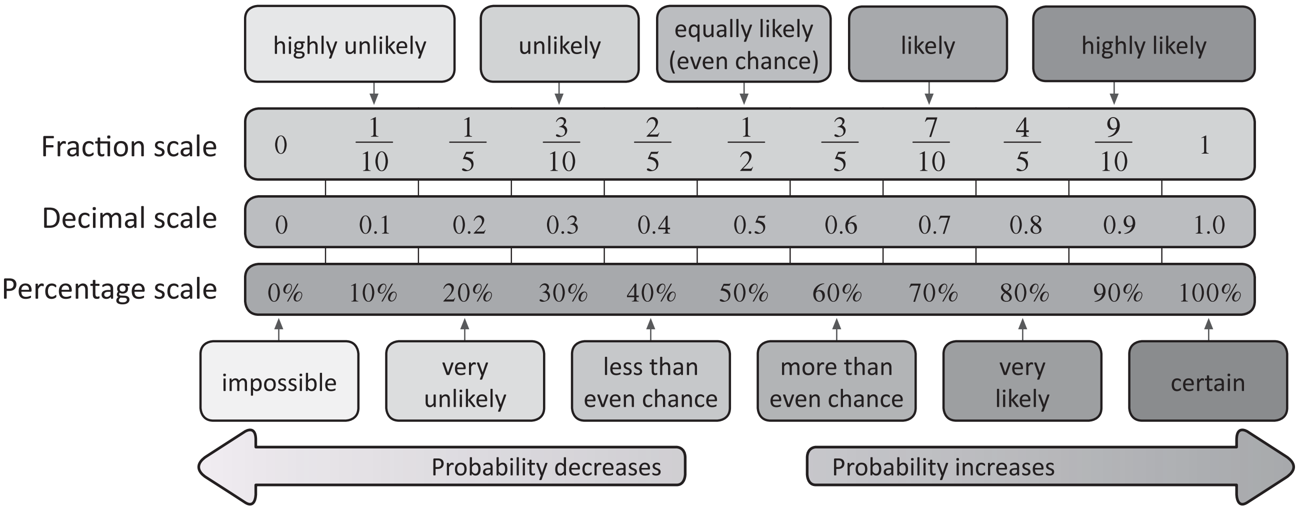
b Calculate $P(No\ ball)$ and $P(Ball)$ when a cup is lifted up as a percentage.

$P(No\ ball) =$ $P(Ball) =$

c Calculate $P(No\ ball) + P(Ball)$.

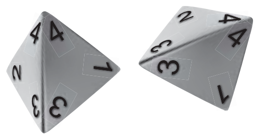
Theoretical probability

Words, fractions, decimals and percentages are all used to describe the **probability** of an outcome. This scale links the words used to describe chance to their **approximate** calculated probability value.



8 All the possible outcomes when two, four-sided dice are rolled are shown in the table below:

Second Die \ First Die				
	1, 1	2, 1	3, 1	4, 1
	1, 2	2, 2	3, 2	4, 2
	1, 3	2, 3	3, 3	4, 3
	1, 4	2, 4	3, 4	4, 4



a Calculate $n(S)$ for the outcomes. $n(S) =$

b Calculate the number of these favourable outcomes:

(i) $n(Even\ sum) =$

(ii) $n(Sum\ of\ 5) =$

(iii) $n(Sum\ of\ 6) =$

(iv) $n(Sum\ of\ 3) =$

(v) $n(Sum < 5) =$

(vi) $n(At\ lease\ one\ 3) =$

(vii) $n(Sum > 2) =$

(viii) $n(Sum\ is\ a\ prime) =$

c Calculate and describe the probability of the favourable outcomes in part **b** occurring.

(i) $P(Even\ sum) =$

$\therefore$ ....

(ii) $P(Sum\ of\ 5) =$

$\therefore$ ....

(iii) $P(Sum\ of\ 6) =$

$\therefore$ ....

(iv) $P(Sum\ of\ 3) =$

$\therefore$ ....

(v) $P(Sum < 5) =$

$\therefore$ ....

(vi) $P(At\ lease\ one\ 3) =$

$\therefore$ ....

(vii) $P(Sum > 2) =$

$\therefore$ ....

(viii) $P(Sum\ is\ a\ prime) =$

$\therefore$ ....

Mathletics Passport © 3P Learning

Complementary events

So far we know that for the probability of an event:

- The total number of favourable outcomes can never be more than $n(S)$.
- The probability of an event $P(E)$ can **only** be any value from $\mathbf{0}$ to $\mathbf{1}$ or $\mathbf{0\%}$ to $\mathbf{100\%}$.

Complementary events in probability are about predicting the chance of the other possible events.

In other words, the probability of a certain event **not** happening.

$$P(Event\ \mathbf{NOT}\ Happening) = P(\overline{Event})$$

$$P(Winning\ a\ game) = P(\overline{Losing\ a\ game})$$

$P(\overline{E})$: The bar drawn over the event means complementary (or not)

Calculate these probabilities and look for a relationship between the two

Axis Airport had six jumbo jets, three airbuses and one helicopter land during a 1 hour period.
A plane spotter outside the airport was looking at what type of planes landed during this 1 hour period.

Calculate:

(i) $P(A\ jumbo\ jet\ was\ spotted) = \dfrac{Total\ number\ of\ jumbo\ jets\ that\ landed = 6}{Total\ number\ of\ aircraft\ that\ landed = 6+3+1 = 10}$

$$= \frac{3}{5}$$

(ii) $P(\overline{A\ jumbo\ jet\ was\ spotted}) = \dfrac{Total\ number\ of\ other\ aircraft\ landed = 3+1}{Total\ number\ of\ aircraft\ that\ landed = 6+3+1 = 10}$

$$= \frac{2}{5}$$

Can you see the relationship between the two probability calculations?

$$P(\overline{A\ jumbo\ jet\ was\ spotted}) = 1 - P(A\ jumbo\ jet\ was\ spotted)$$

$$= 1 - \frac{3}{5}$$

$$= \frac{2}{5}$$

This rule applies to all complementary probabilities:

- $P(\overline{E}) = 1 - P(E)$ when a decimal or fraction.
- $P(\overline{E}) = 100\% - P(E)\%$ when a percentage.

Complementary events

4 Calculate the complementary probability for each of these:

a) $P(Blue) = \frac{1}{3}$

$\therefore P(Not\ blue) =$

b) $P(Good\ reception) = 85\%$

$\therefore P(Poor\ reception) =$

c) $P(Arriving\ on\ time) = 0.30$

$\therefore P(\overline{Arriving\ on\ time}) =$

d) $P(Raining\ tomorrow) = \frac{2}{5}$

$\therefore P(\overline{Raining\ tomorrow}) =$

e) $P(\overline{Parrot\ talking}) = 0.17$

$\therefore P(Parrot\ talking) =$

f) $P(\overline{Have\ green\ eyes}) = 74.4\%$

$\therefore P(Have\ green\ eyes) =$

5 A lucky dip with 50 vouchers contains these possible prizes:

- $30 \times \$5$ mobile phone credit
- $15 \times \$10$ mobile phone credit
- $4 \times \$25$ mobile phone credit
- $1 \times$ new prepaid mobile phone

Write these prize probabilities as simplified fractions if one voucher is chosen randomly.

a) $P(\$25\ mobile\ phone\ credit)$

b) $P(\overline{\$25\ mobile\ phone\ credit})$

c) $P(New\ prepaid\ mobile\ phone)$

d) $P(\overline{New\ prepaid\ mobile\ phone})$

e) $P(\$5\ or\ \$10\ mobile\ phone\ credit)$

f) $P(\overline{\$25\ credit\ or\ a\ new\ prepaid\ mobile\ phone})$

g) Comment on the relationship between the events for parts e and f.

Independent and dependent events

When two or more simple events take place we call it a compound event.

There are two main types of compound events:

- **Independent:** The outcomes of one event does not affect the outcomes of the other.

Eg: Flipping two coins.
The outcome of one coin has no effect on the outcome of the other.

- **Dependent:** The outcome of one event affects the outcome of the other.

Eg: Popping the balloon in a bunch that contains a prize.
The chance of popping the winning balloon increases after each attempt as there are fewer balloons left to pop.

Random selections of two or more objects can happen one of two ways:

- **With replacement:** Each object selected is replaced before making the next selection. The sample space size is the same for all selections, so the outcome of each event is **independent** of the one before.

- **Without replacement:** Each object selected is not replaced before making the next selection. The sample space is smaller for the next selection, so the outcome of each event is **dependent** on what is left after the previous event.

Identify each of these types of events

(i) Tossing a coin and rolling a die (singular of dice).

Independent — Outcome of the coin toss does not affect the outcome on the die

(ii) Selecting two tiles together of the same colour from a bag.

Dependent — Two tiles together is a without replacement selection

(iii) Rolling a pair of normal playing dice.

Independent — Outcome of each die is not affected by the outcome on the other

(iv) An mp3 player selecting two songs by the same artist, one after the other while on random shuffle mode.

Dependent — There are less songs to randomly select from after the first one is played

 © 3P Learning

Independent and dependent events

3 (i) Identify each of these compound events as either dependent or independent.
(ii) Describe how you could change each event into the other type.

a Rolling a die twice and recording the sum.

(i) ☑ Independent ☐ Dependent

(ii) Roll the die twice to record the sum, only if an odd number occurs on the first roll. The second role (and sum) is now dependent on the outcome of the first roll.

b Picking two coloured discs from a bag containing yellow, green and red discs without replacement.

(i) ☐ Independent ☐ Dependent

(ii)

c Guessing the number between 1 and 20 that Vaneeta is thinking of in two or more attempts.

(i) ☐ Independent ☐ Dependent

(ii)

d Recording the colour this spinner stops on each time it is spun and the number rolled on a four sided die.

(i) ☐ Independent ☐ Dependent

(ii)

e Selecting three numbers from a bag at random in descending order with replacement.

(i) ☐ Independent ☐ Dependent

(ii)

f Selecting one key from each of two identical sets of key that will open the same lock.

(i) ☐ Independent ☐ Dependent

(ii)

Mutually exclusive and inclusive events

Mutually exclusive events cannot happen at the same time.

Inclusive events can happen at the same time.

When rolling a die:

- A number that is odd or is a multiple of two cannot happen at the same time.
  ∴ these are mutually exclusive events
- A number that is odd or is a multiple of three can happen at the same time.
  ∴ these are inclusive events (or not mutually exclusive)

Both types of events use the words 'and', 'or', 'either' and 'at least' in probability statements.

- Inclusive and: Where events X and Y can happen.

  Eg: A musician playing the guitar (X) while singing (Y).

- Exclusive-Or: Where **either** event X **or** Y can happen but not at the same time.

  Eg: A person shouting (X) or whispering (Y).

- Inclusive-Or : Where events X or Y or both X and Y can happen.

  Eg: Jenny shaking hands (X) or Linda shaking hands (Y).
  (X **and** Y is Jenny and Linda shaking hands with each other or other people).

'At least' is used for inclusive-or statements. Because 'at least' means **either** X **or** Y **or** both X **and** Y:
'At least **either** events X **or** Y **or** both occurring'.

Write the type of exclusive and inclusive events each of these statements represent

(i) Picking one disc which is either blue or red from a bag containing red, blue and green discs.

Mutually Exclusive (Exclusive-Or): Blue or Red (but not both) colours can be selected.

(ii) Picking two discs, one blue and the other red, from a bag containing 3 red and 3 blue discs.

Inclusive-and: Picking a disc of each colour can happen at the same time.

(iii) Catching at least one of the 10 tadpoles in a pond using a net.

Inclusive-Or: One, two, three or more can be caught. A minimum of one must be caught.

(iv) An outcome of only one Tail when flipping three coins.

Mutually Exclusive (Exclusive-Or): Only coin 1, coin 2 or coin 3 can be a Tail, not a combination of this.

Mutually exclusive and inclusive events

1. Decide if these are mutually exclusive or inclusive events by ticking the right term.

   a. Flipping a Head or Tail on two different coins.

      ☐ Mutually exclusive ☐ Inclusive

   b. A light switch in the 'on' or 'off' position.

      ☐ Mutually exclusive ☐ Inclusive

   c. Winning first or second prize in a local raffle with one ticket.

      ☐ Mutually exclusive ☐ Inclusive

   d. Winning first or second prize in a local raffle with two tickets.

      ☐ Mutually exclusive ☐ Inclusive

2. A bag contains ten cards and each card has a different number (from 0 through to 9) written on it. Describe mutually exclusive and inclusive events that involve randomly selecting a card from this bag.

   a. Mutually exclusive events:

   b. Inclusive events:

3. Two boxes contain the following:

   Box A: 1 orange, 1 blue and 5 yellow marbles.
   Box B: 1 yellow, 1 green and 3 black marbles.

   Describe two different (a) mutually exclusive and (b) inclusive events that involve randomly selecting two marbles (with or without replacement) from the same box, or one from each box contain the following:

   a. Mutually exclusive events:

   b. Inclusive events:

Mutually exclusive and inclusive events

4 Tick the correct type of exclusive or inclusive events each of these statements represent.

a. A student selected from the class has either brown hair or brown eyes.

☐ Exclusive Or ☐ Inclusive Or ☐ Inclusive And

b. Dropping a cup and spilling all the contents.

☐ Exclusive Or ☐ Inclusive Or ☐ Inclusive And

c. One of two teachers selected randomly in a school catches public transport to school.

☐ Exclusive Or ☐ Inclusive Or ☐ Inclusive And

d. Boiling and freezing a container of water.

☐ Exclusive Or ☐ Inclusive Or ☐ Inclusive And

e. A person selected at random is either sitting down or standing up.

☐ Exclusive Or ☐ Inclusive Or ☐ Inclusive And

f. Rolling a number larger than 5 and an even number on a normal 6-sided die.

☐ Exclusive Or ☐ Inclusive Or ☐ Inclusive And

g. Spelling a word correctly and using it properly in a sentence.

☐ Exclusive Or ☐ Inclusive Or ☐ Inclusive And

h. Selecting a red card and the number 7 from a normal pack of playing cards

☐ Exclusive Or ☐ Inclusive Or ☐ Inclusive And

i. A student selected randomly during period 3 was doing Physical Education or Music.

☐ Exclusive Or ☐ Inclusive Or ☐ Inclusive And

5 Earn yourself and AWESOME passport stamp with this one.
Professor Probability visits one day and during a chat exclaims:

"There is no such thing as a single inclusive, dependent event!"

Explain why you agree or disagree with the statement and give an example to support your answer.

Two-way table probabilities

The fraction of observed results in any experiment/collection of trials is called the relative frequency.

Number of times it happens ↓

$$\text{Relative frequency} = \frac{\text{The frequency of the outcome being observed}}{\text{The number of trials completed}}$$

The values along with the row and column totals make relative frequency calculations easy.

Complete the two-way table and use it to answer the given questions below

After tossing a coin and rolling a four-sided die together 100 times, the tally of each outcome pairing was recorded.

$(H, 1) =$ 卌 卌 III $(H, 2) =$ 卌 III $(H, 3) =$ 卌 卌 II $(H, 4) =$ 卌 卌 IIII

$(T, 1) =$ 卌 卌 卌 $(T, 2) =$ 卌 卌 IIII $(T, 3) =$ 卌 III $(T, 4) =$ 卌 卌 卌 I

(i) Record the observed results into a two-way table.

		4 sided die				
		1	2	3	4	**Total**
Coin	Hand (H)	13	8	12	14	47 ← Total heads
	Tail (T)	15	14	8	16	53 ← Total tails
	Total	28	22	20	30	100 ← Sum of the column/row totals should both equal the total number of trials.
		↑ Total 1s	↑ Total 2s	↑ Total 3s	↑ Total 4s	

(ii) Calculate the relative frequency and theoretical probability for the outcome $(T, 4)$.

Relative Frequency (or Experimental Probability)

$$n(T, 4) = 16, \quad n(Trials) = 100$$

$$\text{Relative frequency of } (T,4) = \frac{n(T,4)}{n(Trials)} = \frac{16}{100} = \frac{4}{25}$$

Theoretical Probability

$$n(T, 4) = 1, \quad n(S) = 8$$

$$P(T,4) = \frac{n(T,4)}{n(S)} = \frac{1}{8}$$

The more trials completed, the closer we expect the relative frequency to match the theoretical probability.

(iii) Calculate the relative frequency for flipping a Head when the number 3 was rolled.

$$n(H, 3) = 12, \quad n(Trials\ in\ which\ a\ 3\ was\ rolled) = 20$$

$$\text{Relative frequency of heads when a 3 is rolled} = \frac{n(H,3)}{n(Trials\ in\ which\ a\ 3\ was\ rolled)}$$

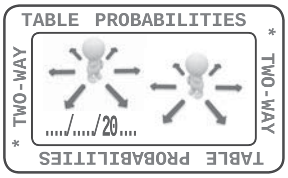
$$= \frac{12}{20}$$

$$= \frac{3}{5} \quad \text{Simplified form}$$

$$= 60\% \quad \text{Percentage form}$$

Mathletics Passport © 3P Learning

Two-way table probabilities



3 This two-way table shows the results of a random survey of students in a school who were asked a yes/no question followed by a multiple choice question.

		Q2				
		A	B	C	D	**Total**
Q1	Yes (Y)	4	7	1	0	12
	No (N)	8	17	10	3	38
	Total	12	24	11	3	50

a How many students were surveyed in this school?

b How many students surveyed selected 'C' for Question 2?

c What was the most common outcome for the two questions asked in this survey?

d What outcome did not occur for the two questions asked in this survey?

e What is the frequency for the outcome 'Yes, A'?

f What is the relative frequency for the outcome 'Yes, A'?

g What is the relative frequency for an answer of 'No' to Q1 as a percentage?

4 Numbers 1 through to 20 were printed on two packs of twenty cards. One pack printed using red ink (R) and the other printed using green ink (G). The two packs were then shuffled together.

a Twenty four cards were randomly selected (with replacement) and the outcomes recorded. Complete the two-way table given using the recorded outcomes below.

$(G, 15)$	$(G, 1)$	$(G, 11)$	$(R, 6)$
$(R, 5)$	$(R, 18)$	$(R, 3)$	$(G, 17)$
$(R, 12)$	$(G, 8)$	$(R, 19)$	$(R, 2)$
$(G, 7)$	$(R, 1)$	$(G, 3)$	$(G, 6)$
$(G, 10)$	$(R, 3)$	$(G, 5)$	$(G, 6)$
$(R, 14)$	$(R, 16)$	$(G, 13)$	$(R, 5)$

		Colour		
		Red	Green	**Total**
Number	≤ 10			
	>10			
	Total			

b Calculate the expected (theoretical) probability of selecting a green card with a number ≤ 10 and its relative frequency following the random selections.

c If the number of random selections is increased greatly, what do you expect will happen to the theoretical probability and relative frequency values?

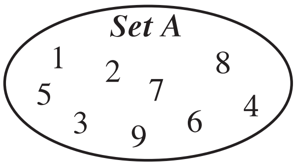
Set diagram basics

Venn diagrams show the members of collected data arranged into groups called sets.
They show what members are unique to a set and which ones occur in more than one set.

All data – including anything outside of the sets – are members of the Universal Set (U).

For example:

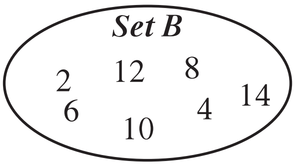
Set A: All intergers from 1 to 9



$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ Set notation

$n(A) = 9$ Number of members

Set B: All even integers between 1 and 15

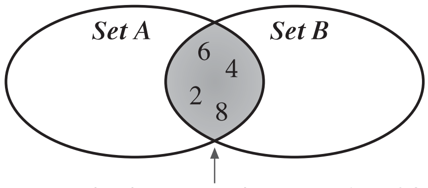


$A = \{2, 4, 6, 8, 10, 12, 14\}$ Set notation

$n(B) = 7$ Number of members

Intersection ($\cap$)

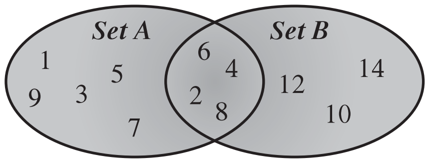
- Where Sets A **and** B overlap.
- So, $A \cap B = \{2, 4, 6, 8\}$ and $n(A \cap B) = 4$.



These are both integers between 1 and 9 AND even integers between 1 and 15

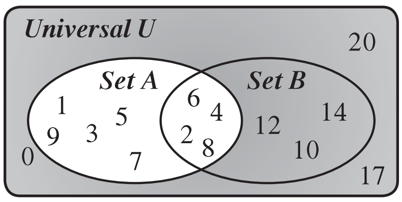
Union ($\cup$)

- The members that are in either set A **or** set B **or** both.
- So, $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14\}$ and $n(A \cup B) = 12$.



Complement ($\overline{A}$)

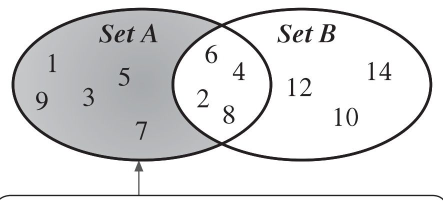
- Everthing that is **not** in the set.
- So, $\overline{A} = \{0, 10, 12, 14, 17, 20\}$ and $n(\overline{A}) = 6$. Also $\overline{B} = \{0, 1, 3, 5, 7, 9, 17, 20\}$ and $n(\overline{B}) = 8$.
- $A^c, \overline{A}, A', \tilde{A}, A^{\sim}$ are all different ways used to show the complement of set A.



Psst: Three numbers (0, 17, 20) were added to the Universal set that don't fit into sets A or B.

Difference ($-$)

- The difference (also called the relative complement) is used to find what is unique to a set.

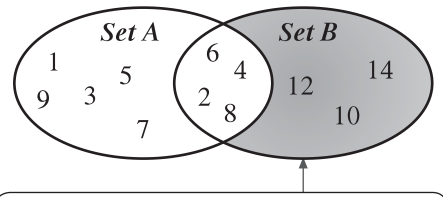


$A - B$ means Set A but **not** Set B

$\therefore A - B = \{1, 3, 5, 7, 9\}$

$n(A - B) = 5$.

The order is important



$B - A$ means Set B but **not** Set A

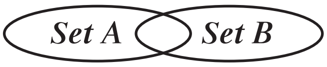
$\therefore B - A = \{10, 12, 14\}$

$n(B - A) = 3$.

Set diagram basics

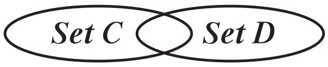
1 (i) Shade each of these diagrams to match the statements given.
(ii) Write the shaded area using the symbols ∩, ∪, – or $\overline{Set}$.

a (i) All the data in both sets



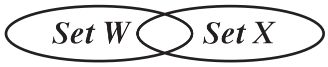
(ii)

b (i) All members in Set C



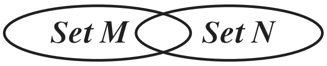
(ii)

c (i) Data is shared by both sets



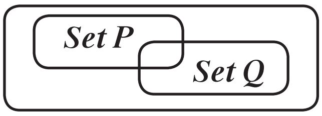
(ii)

d (i) Set N but not Set M



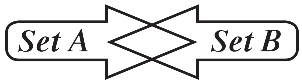
(ii)

e (i) Everything except Set Q



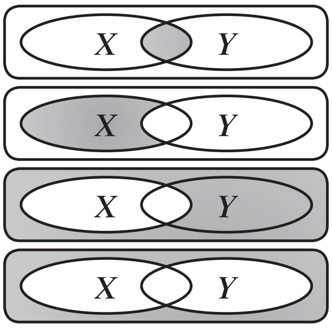
(ii)

f (i) The members of Set B that are not unique to Set B

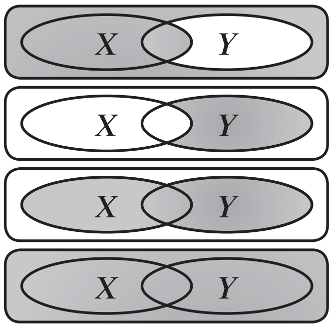


(ii)

2 Match these Venn diagrams with the correct description in the middle.



- X but not Y
- Y but not X
- X or Y or both
- X and Y
- neither X or Y
- not X
- not Y
- all data



3 A box contains ten balls, all numbered from 1 to 10.

Set A = balls containing multiples of 3, Set B = balls containing odd numbers.

a Write in the members of each set below:

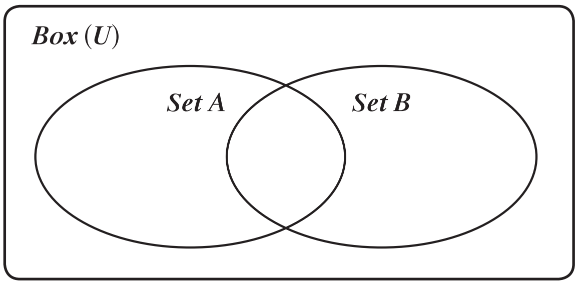
(i) $A = \{$ 3, 6, 9 $\}$

(ii) $B = \{\quad\}$

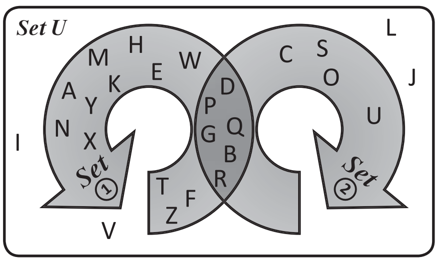
(iii) $U = \{\quad\}$

(iv) $A \cap B = \{\quad\}$

b Put the data into this Venn diagram.



4 Write down the members of the following sets displayed in the Venn diagram below:



(i) ① =

(ii) ① ∩ ② =

(iii) $\overline{②}$ =

(iv) ② – ① =

(v) ① – ② =